Magnetic field of atomic nuclei

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In this paper we evaluate typical values of magnetic field energy and electromagnetic angular momentum of atomic nuclei with very simple scenarios, for the first time. We point out possible situations in which magnetic energy and angular momentum might play essential roles in nuclear physics.

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I. INTRODUCTION

Electric field is very important in atomic nuclei. It is the Coulomb repulsive force that leads to the fact that the number of elements is limited. The energy of electric field is one essential component of nuclear binding energy (in particular for heavy mass nuclei) [1]. On the other hand, the role of magnetic field has been by far less investigated in nuclear physics hitherto.

It is therefore one of the purposes in this paper to evaluate both the magnetic energy and electromagnetic angular momentum in atomic nuclei. The results of angular momentum are easily generalized to the case of one charge particle such as a proton or an electron in magnetic field, in which case the angular momentum are much larger than 1 $\hbar$ if the space is large or magnetic field is strong enough.

This paper is organized as follows. In Sec. II we investigate the magnetic energy of nuclei. In Section III we study the electromagnetic angular momentum. We summarize this paper in Sec. IV.

II. MAGNETIC ENERGY

We first exemplify magnetic energy by deutron, which is the weakly bound state of a proton and a neutron. One quickly get a crude panoramic view with this simple case. Deutron is reasonably approximated to be a proton and a neutron in s orbit with spins aligned. Under this assumption, magnetic interaction energy between the proton and the neutron as below.

$$W_{m_1m_2} = \frac{\mu_0}{4\pi}\left(\frac{\vec{m}_1\cdot\vec{m}_2 - 3(\vec{m}_1\cdot\vec{e}_r)(\vec{m}_2\cdot\vec{e}_r)}{r^3}\right) , \quad (1)$$

where μ_0 is the permeability of vacuum, and $\vec{m}_1$ and $\vec{m}_2$ are magnetic moments of respectively a free proton and free neutron, and r is the distance between proton and neutron. We assume that both spins are in parallel with $\vec{e}_r$, and we have

$$W_{m_1m_2} = \frac{\mu_0}{4\pi}\frac{2|\mu_{\rm p}\mu_{\rm n}|}{r^3} \simeq 0.0027 \text{ MeV}, \quad (2)$$

One sees that the magnetic interaction energy between a neutron and a proton in deutron is in the order of keV. We note that this energy does not include the "self" energy of individual nucleon, for which we shall use a simple model to evaluate later in this paper.

For nuclei with many protons and neutrons, we simplify the entire nuclei as one uniformly magnetized ball, with the total magnetic moment $\vec{m}$ and a radius R_0. In this case the distribution of magnetic induction $\vec{B}(\vec{r})$ is as follows [2]. Inside the nucleus,

$$\vec{B}(\vec{r}) = \frac{2}{3}\mu_0\vec{M}_0 , \quad (3)$$

where $\vec{M}_0 = M_0\vec{e}_z$ is magnetization strength (constant), with

$$\vec{m} = \frac{4}{3}\pi R_0^3 M_0\vec{e}_z ,$$

Correspondingly, the magnetic energy inside (denoted by $W_{\rm i}$) is given by

$$W_{\rm i} = \frac{4\pi R_0^3}{3}\frac{1}{2\mu_0}\left(\frac{2}{3}\mu_0 M_0\right)^2 = \frac{\mu_0\vec{m}^2}{6\pi R_0^3} .$$

Outside the nucleus [2],

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi r^3}\left(\frac{3(\vec{m}\cdot\vec{e}_r)\vec{e}_r - \vec{m}}{r^3}\right) , \quad (4)$$

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where r is the space coordinate with respect to the center of nucleus. By using the following identity

$$\begin{aligned}[3(\vec{m}\cdot\vec{e}_r))\vec{e}_r-\vec{m}]^2 &= m^2[(2\cos\theta)^2+\sin^2\theta] \\ &= m^2(3\cos^2\theta+1)\ ,\end{aligned}$$

where θ is the angle between $\vec{e}_z$ and $\vec{e}_r$. By integrating over the full space, one obtains the magnetic energy outside the nucleus (denoted by W_e) as below.

$$\begin{aligned}W_\mathrm{e} &= \int \mathrm{d}\tau\frac{\vec{B}^2}{2\mu_0}=\int \mathrm{d}\tau\frac{\mu_0}{32\pi^2}\cdot\frac{\vec{m}^2(3\cos^2\theta+1)}{r^6} \\ &= \frac{\mu_0\vec{m}^2}{12\pi R_0^3}\ .\end{aligned}$$

Summing up W_i and W_e, one obtain the total magnetic energy W

$$W=W_\mathrm{i}+W_\mathrm{e}=\frac{\mu_0\vec{m}^2}{4\pi R_0^3}\ . \tag{5}$$

According to Eq.(5), the value of W is proportional to R_0^{-3} or approximately $1/A$. As the magnetic moments do not increase with A (typically a few μ_N), the value of W is larger for smaller A. This energy is the largest for proton: if one takes $m=2.793\mu_N$ and $R_0=0.84$ fm (charge radius), the value of W is 210 keV, according to this oversimplified classical model.

Now we present typical values of W for a few light nuclei, by using the NNDC database [3] for magnetic moments and Ref. [4] for charge radii, in this classical model. The value of magnetic energy is 27 keV for ^{3}H ($m=2.98\ \mu_N$ and $R=1.76$ fm), 9.5 keV for ^{3}He ($m=2.13\ \mu_N$ and $R=1.97$ fm) and 12 keV for ^{7}Li ($m=3.26\ \mu_N$ and $R=2.44$ fm), 11 keV for ^{41}Sc ($m=5.43\ \mu_N$ and $R=3.50$ fm). In most cases, the values of W is only a few keV or even well below. Currently, the uncertainty of nuclear mass formulas is well above 100 keV, thus the contribution from magnetic energy is not essential at all, unless nuclear mass models could achieve the accuracy of a few keV in future.

III. ELECTROMAGNETIC ANGULAR MOMENTUM

Atomic nuclei have strong electric field. With further considering the magnetic moment, magnetic momentum density is given by $\epsilon_0\vec{E}\times\vec{B}$, which is easily seen to yield an angular momentum in parallel with magnetic moment. Suppose that a given nucleus that we consider is (approximately) spherical with electric charge uniformly distributed. We denote the proton number of this nucleus by using Z, and have the electric field strength

$$\vec{E}(\vec{r})=\frac{Ze}{4\pi\epsilon_0 R_0^3}\vec{r}$$

inside the nucleus, and

$$\vec{E}(\vec{r})=\frac{Ze}{4\pi\epsilon_0 r^3}\vec{r}$$

outside the nucleus. The resultant angular momentum density is

$$\vec{l}=\vec{r}\times(\epsilon_0\vec{E}\times\vec{B})\ ,$$

where $\vec{B}$ is given in eqs. (3) and (4). Integrating $\vec{l}$ in the full space, one obtains the total electromagnetic momentum $\vec{L}$ of the system, as below.

$$\begin{aligned}\vec{L} &= \int \mathrm{d}\tau\,\vec{l}=\int_0^{R_0} r^2\mathrm{d}r\mathrm{d}\Omega\ [\vec{r}\times(\epsilon_0\vec{E}\times\vec{B})]+\int_{R_0}^{\infty} r^2\mathrm{d}r\mathrm{d}\Omega\ [\vec{r}\times(\epsilon_0\vec{E}\times\vec{B})] \\ &= \int_0^{R_0} r^2\mathrm{d}r\mathrm{d}\Omega\ \vec{r}\times\left(\epsilon_0\frac{Ze}{4\pi\epsilon_0 R_0^3}\vec{r}\times\frac{2}{3}\mu_0 M_0\vec{e}_z\right)+\int_{R_0}^{\infty} r^2\mathrm{d}r\mathrm{d}\Omega\ \vec{r}\times\left[\epsilon_0\frac{Ze}{4\pi\epsilon_0 r^3}\vec{r}\times\frac{\mu_0}{4\pi r^3}\left(\frac{3(\vec{m}\cdot\vec{e}_r)\vec{e}_r-\vec{m}}{r^3}\right)\right] \\ &= \int_0^{R_0} r^2\mathrm{d}r\mathrm{d}\Omega\ \vec{r}\times\left(\epsilon_0\frac{Ze}{4\pi\epsilon_0 R_0^3}\vec{r}\times\frac{\mu_0\vec{m}}{2\pi R_0^3}\right)-\int_{R_0}^{\infty}\mathrm{d}r\mathrm{d}\Omega\ \vec{r}\times\left[\frac{\mu_0 Ze}{16\pi^2 r^4}(\vec{r}\times\vec{m})\right]\ .\end{aligned}$$

The axial symmetry of the system yields that $\vec{L}=L_z\vec{e}_z$. One has

$$L_z = -\frac{\mu_0 Ze}{8\pi^2 R_0^6}m\int_0^{R_0} r^4\mathrm{d}r\sin^2\theta\mathrm{d}\Omega+\int_{R_0}^{\infty}\frac{\mu_0 Ze}{16\pi^2 r^2}m\mathrm{d}r\sin^2\theta\mathrm{d}\Omega=\frac{\mu_0 Zem}{10\pi R_0}\ .$$

For typical nuclei, the value of L such evaluated is found to be very small; for example, for the ^{41}Sc nucleus, $m=5.43\mu_N$ and $R_0=3.50$ fm, this angular momen-

tum is only 1.0×10^{-2} $\hbar$; and for proton, the value of $L \simeq 1.0 \times 10^{-3}$ $\hbar$. Because orbital angular momentum of given system is always quantized in forms of multiple $\hbar$, we conclude that electromagnetic angular momentum of atomic nuclei *is actually zero*.

It is now interesting to consider another simple system, a point charged particle in uniform magnetic field $\vec{B} = B_0\vec{e}_z$. In this case the electromagnetic angular momentum is simply given by

$$\vec{L} = \int d\tau \ \vec{r} \times \left(\epsilon_0 \frac{Ze}{4\pi\epsilon_0 r^3}\vec{r} \times B_0\vec{e}_z\right) = L_z\vec{e}_z \ ,$$

with

$$L_z = -\frac{ZeB_0}{3}r^2 \ , \tag{6}$$

where Ze is the charge of the point particle. If both r (i.e., the space to be considered) and B_0 are not very large, L is well below 1 $\hbar$.

On the other hand, according to Eq. (6), the value of $\vec{L}$ increases with r and magnetic field strength $\vec{B}_0$. In case that B_0 is taken to be very large under an extreme condition, e.g., in neutron star, the value of L in Eq. (6) could become much larger than 1 $\hbar$ and play an essential role in certain processes related to angular momentum of nucleons and electrons in neutron star.

Another possible application of Eq. (6) is the relativistic heavy ion collision, in which the magnetic field is in parallel with the orbital angular momentum. Because electromagnetic angular momentum of the system $\vec{L}$ is antiparallel to the magnetic field $\vec{B}$, as shown in Eq. (6), the emergence of electromagnetic angular momentum $\vec{L}$ suddenly enhances the value of orbital angular momentum of the whole system during the collision, *if the colliding fire-ball is not electric neutral*. The value of $\vec{L}$ could be as large as 10^3 $\hbar$ ($\vec{L}$ is proportional to the strength of the magnetic field) or even larger, depending on the number of nucleons in the spectator part and the colliding energy; clearly, the larger the colliding energy and the number of protons in the spectator of collision, the larger the value of $\vec{L}$ is. This provides us with a possible mechanism and view of the fact that fire-ball of relativistic heavy ion collision becomes more and more electric neutral with the colliding energy.

IV. SUMMARY

To summarize, in this paper we investigate, for the first time, as far as we know, magnetic energy and eletromagnetic angular momentum of atomic nuclei with very simple assumption of eletromagnetic field. All these quantities are obtained in compact formulas.

We present a few typical values of magnetic energy and electromagnetic angular momentum for atomic nuclei, with a note that the value of magnetic energy is negligibly small in current nuclear mass models, and that the value of eletromagnetic angular momentum is actually zero for conventional atomic nuclei. On the other hand, it is also worthy to point out that the value of electromagnetic angular momentum for a nucleon might be larger than 1 $\hbar$ and might play an important in physics under certain extreme conditions such as in neutron stars with very strong magnetic field. This might be one of the mechanisms of the fact that fire-ball of RHIC is electric neutral.

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